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The Methodology of Teaching a Logarithmic Slide Rule in Historical Sequence

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A few years after the publication of the first logarithm table in the year 1614 by John Napier the spreading of a computing aid begins, which during 350 years has produced a variety of variants for numerous applications. The history of this instrument, the logarithmic slide rule, is well documented and does not have to be repeated here.

The independent use of the slide rule requires that the user has been instructed before using. In the analysis of numerous historical sources on this subject, changes in the teaching method can be observed, which cannot be localized exactly in terms of time but can be clearly identified.

The historical methods and their changes over time can be divided into three periods. They are described in more detail below with the help of a selection of printed instructions. There are two main questions: how did the authors instruct the trainees in using the slide rule, and did they name the basics of the instrument? Users initially apply a tool the way they learned to use the tool. Therefore knowledge of the historical methodology can give us a look back on the handling of the slide rule in past centuries.

The First Period

The beginning of the first period is marked by the first description of a two-sided logarithmic instrument with movable scales, a so-called sliding gunter, by Seth Partridge 1658.[1] The author first explains the scales and their arrangement.[2] Reading examples are not given, although they are essential for an unschooled person. Because the author asks the reader if he can also carry out the calculations with compasses on logarithmic scales, so-called gunter lines, one must assume that he presumes knowledge about their use and thus their reading. Similarly, any reference to logarithms as the theoretical basis of the scales and their use is missing. Later authors include the reading of the numbers and their ambiguity in value on a logarithmically divided scale in their lessons.

One of the computational examples at Partridge describes the calculation of the circumference of a circle with a given diameter (Partridge 1661, page 47):

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The Diameter of a circle being given,
to find the Circumference.
The Analogie stands thus:
As 1 is to the Diameter, So is 3,142
to the Circumference. Or,
As 7 to 22; So is the Diameter to
the Circumference. If the Diameter
be 15 inches, what is the
Circumference? Set 1 on the first,
to 15 on the second, and then right
angainst 3,142 on the first, is
47,13 on the second: So much is the
Circumference of that circle. Or
Set 7 on the first, to 22 on the
second, and then against 15 on the
first, is 47,13 on the second, as
before.
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First and *second* are the two opposing scales. That the use of the slide rule is taught step by step in a sequence of settings and readings is notable. This procedure is applied to all sample applications. A counterpart is in contemporary arithmetic books for practitioners, which likewise give no explanations or derivations.

With a plurality of application examples, 140 in 13 chapters on geometry, trigonometry, navigation, mechanics, and other subjects, Partridge tries to convince the reader that the instrument is suitable for all computationally solvable applications. The given instructions for use refer only to specific applications; they can even repeat themselves in other sections. Furthermore, all calculations are based on proportions, even if in simple calculations such as the multiplication in the above example one of the factors has the value 1. This approach is found in the title of the work, and is counteracting a characteristic feature of the slide rule, namely, the ability to display numbers of equal proportions in one setting. This approach also corresponds to the contemporary representation of numerical dependencies, which is replaced by the view as a function since the middle of the 18th century.

In the above example that the approximation[3] 22/7, which is already known from ancient times, can be set on the scales more easily with two integers than with the also mentioned decimal fraction 3.142 is noteworthy.

In his work, Partridge gives no indication of the theory underlying the instrument, and therefore does not mention logarithms and their inventor.

Partridge is comparable to Michael Scheffelt. In the second edition of his book on his new invention[4] from 1718, he presents an arrangement with two scales, which can be moved against one another, in addition to the rod with a logarithmically divided scale, which can only be used by means of compasses. Scheffelt, like Partridge, is bound in thinking in proportions, which he also expresses in the title of his book:

> Neu-erfundener Maß-Stab / Auf welchem Alle Proportiones der gantzen Mathesis ohne mühsames Rechnen, so wohl mit- als ohne Hand-Circul, in Arithmetica, Geometria [...] können gesucht und gefunden werden.
>
> *(Newly invented measuring-rod / on which all proportions of the whole mathematics without laborious calculations, with and without compasses, in arithmetic, geometry […] can be searched and found.)*

In numerous exercises Scheffelt demonstrates the possible applications of this arrangement. His instructions for the calculation of circumference and area of a circle are (Scheffelt, 1718, page 85):

> 56. Wie soll der Inhalt eines Circuls gefunden werden?
> E.g. Es werde gegeben der Diameter eines Circuls **ab** 2°1'. Ist die Frag nach dessen Inhalt? [...]
> Oder ich stelle 7. rechter Hand / zu 22. lincker Hand / und sehe bey 21. rechter Hand / finde 6°6'. lincker Hand die Circumferenz deß Circuls [...]
> Oder ich stelle 4. rechter Hand / zu 21. lincker Hand / und schaue bey 1. rechter Hand / finde lincker Hand / 525. nehme alsdann die Weite von 1. biß 5'2''5'''. stelle solche auß 6°6'. über sich / finde bey nahem 3°4'7''. oder 3°4'6''5'''. den Quadrat- oder flachen Inhalt deß Circuls.[5]
>
> *(56. How o find the area of a circle?*
> *For example. The diameter of a circle **ab** will be given with 2°1‘. The area is asked? [...]*
> *Or I set 7 right hand / to 22 left hand / and look at 21 right hand / find 6°6' left hand / the circumference of the circle [...]*
> *Or I set 4 right hand / to 21 left hand / and look at 1 right hand / find left hand / 525 next take the width from 1 to 5'2''5'' set it from 6°6' beginning / find nearly 3°4'7'' or 3°4'6''5''' the area of the circle).*

Left hand and right hand define the two logarithmic scales according to the direction in which they are moved. For the calculation of the circular area several solutions are shown, above only one is cited. Scheffelt does not address the basis of logarithmic calculation, and of logarithms only insofar as he lists them in a table for the preparation of the logarithmic scale.

Over time the use of the slide rule is extended to an increasing number of applications in the fields of mathematics, geometry, mechanics, and technology. Simultaneously one tries to simplify its application with pre-calculated partial results. An example of this can be found at Joseph Howe 1845.[6]

In the appendix the author names himself *Teacher of the Slide Rule, Hyde, near Manchester*. His instructions are still verbalized at a time when other authors already choose the graphical method. His book contains tabular lists with constant numerical values, which he uses in certain applications.

To find the area of an equilateral triangle, and regular polygons, from 5 to 12 sides.

No of sides.	GAUGE POINTS
3	,433
5	1,720
6	2,598
7	3,634
8	4,828
9	6,182
10	7,694
11	9,366
12	11,196

RULE.

As the guage point upon C is to unity upon D, so is the length of one side or any pologon upon D to the content or area upon C? or set the gauge point upon C to 1 or 10 upon D; then against the length of one side of the polygon upon D, is the content or area upon C.

FIGURE 1: Howe page 26

These tabular lists are either simple multipliers or divisors or the results of complex pre-computed terms. The users are relieved of calculations in between, they even do not need to know their basis or how to calculate them. Howe uses these constants in the calculation of the areas of polygons as in Figure 1[7], in the determination of the weight of solid objects with given dimensions and made from different materials, or in the calculation of the piston diameter of steam engines that are intended to drive water pumps with predetermined dimensions (See Figure 2).[8]

Shortly afterwards Sedlaczek 1856[9] provides a more extensive table for working with polygons using decimal fractions and proportions.

The use of gauge points was not a new invention. Already Everard 1684[10] taught their use. Later manufacturers provide appropriate markings on the scale.[11] New at that time is the steadily growing amount of pre-calculated constants, arranged in tables, so that even more applications are covered. In the course of industrialization the scopes of technical mechanics and machine design are mainly part of these tables.

31

PUMPING ENGINES.

The two following tables of gauge points are for finding the diameters of steam engine cylinders, that will work pumps from 3 to 30 inches diameter, and at given depth in yards. The first table loads its cylinders with 10lbs. upon every square inch of the area in their pistons, and the second table is calculated so as to load the different cylinders with 7lbs. weight upon every square inch, in the area of their pistons.

Table of 10lbs. to the inch.				Table of 7lbs. to the inch.			
Diam of pump	Gauge Point.	Diam of pump	Gauge Point.	Diam of pump	Gauge Point.	Diam of pump	Gauge Point.
3	115	17	367	3	165	17	528
4	204	18	41	4	292	18	591
5	318	19	46	5	457	19	695
6	458	20	51	6	66	20	731
7	625	21	562	7	89	21	81
8	815	22	616	8	117	22	885
9	103	23	67	9	148	23	97
10	127	24	732	10	183	24	406
11	154	25	795	11	222	25	114
12	183	26	86	12	264	26	124
13	2125	27	928	13	808	27	134
14	25	28	99	14	358	28	143
15	2875	29	107	15	412	29	154
16	327	30	115	16	468	30	165

In making use of the two preceding tables, observe the following rule:

As a gauge point on A is to unity on B, so is the length of a column of water in yards on C, to the diameter of the steam cylinder on D; or set unity on B to the gauge point on A, then against any length of a column of water in yards on C is the diameter of a steam cylinder that will work the pump on D.

FIGURE 2: Howe page 31

The Second Period

In the first decades of the 19th century the methodology changed into the second section. Now the hitherto verbal instructions are replaced by simplified graphical representations of the scales. Examples can be found at John Farey 1827, Charles Hoare 1868, as well as Adam Burg 1830, and Leopold Schulz of Straßnicki 1843.[12] Both Burg and Schulz von Straßnicki were supporters of the slide rule in Germany and Austria. Schulz von Straßnicki gives not less than 388 rules for calculations of all kind, including conversions of weights and measures[13] (See Figure 3). The latter became essential during the conversion from old units of measurement to the new metric system in the course of the 19th century in Europe.

Even at the beginning of the 20th century, the company A. W. Faber shows pictures of slide rules with fully divided scales in order to clarify the settings.

170

D. Stereometrie.

337. Regel. A 1 Flächeninhalt der Basis
B Höhe Körperinhalt des Prisma

338. Regel. A 144 Basis in Quadratzollen
B Länge in Fußen Körperinhalt in Kubikschuh

349. Regel. A 36 Basis in Quadratschuh
B Länge in Klaftern Körperinhalt in Kubikklaft.

340. Regel. A 4·32 Basis in Quadratzoll
B Höhe in Schuhen Volumen in Kubikschuh für Pyramide und Kegel

341. Regel. A 3 Basis
B Höhe Volumen der Pyramide oder des Kegels

342. Regel. C Höhe Körperinhalt des Cylinders
D 3·544 Umfang

343. Regel. C Höhe Volumen des Cylinders
D 1·128 Durchmesser

344 Regel. C Höhe in Fußen Volumen des Cylinders
D 13·54 Durchmesser in Zollen

345. Regel. C Höhe Volumen des Kegels
D 1·95 Durchmesser

346. Regel. C Durchmesser der Kugel Volumen der Kugel
D 1·38 Durchmesser

FIGURE 3 : Schulz von Straßnicki, page 170 (calculaion of volume)

In 1868 Hoare reduced the representation of the scales to the essentials (See Figure 4).

9. Circumference and Diameter of Circle (on A and B).

A Set 22 Line of circumferences
B To 7 Line of diameters.

FIGURE 4: Hoare, page 18

In contrast to verbal instructions, graphical representations have an advantage that they are easier to understand and in addition allow the exchange of input and output variables. In the settings shown constant values have already been incorporated. They are given without any explanation. Formulas for the respective calculations are not listed, the theory of the instrument is not mentioned.

We still find a collection of complete instructions, arranged according to subject areas that try to cover as many different applications as possible. This implies that the user did not necessarily had to have relevant knowledge. From this fact, the question arises as to how the user behaved when he had to perform a calculation that is not listed in the collection. In that case, did he look for a similar solution or did he solve the problem by composing partial solutions?

For constant values, there are two types of representation: either as a fractional number, as in Schulz von Straßnicki 1843, or as a proportion of two numbers, as in Cox 1891 in his list with conversion factors for length units.[14]

METRIC SYSTEM.

26 = Inches.	82 = Feet.
66 = Centimetres.	25 = Metres.
82 = Yards.	87 = Miles.
75 = Metres.	140 = Kilometres.
4300 = Links.	43 = Chains.
865 = Metres.	865 = Metres.

FIGURE 5: Cox, page 20, Extract

Explanations for constant numbers, which are not immediately obvious, are rare. Adam Burg, already mentioned above, is one of the few authors who explain the derivation of constants in detail. He also preferably uses proportions with integers instead of decimal fractions, as with 109/123 or 39/44 for sqrt (pi/4) and 55/70 or 95/121 for pi/4.

The Third Period

In the last decades of the 19th and in the first decades of the 20th century, methodology again turned into a third section. Predefined instructions for setting the slide rule are replaced by instructions for a generalized usage. As usual, the authors demonstrate simple basic tasks such as multiplication, division, roots, and powers. They present a calculating example as a formula, and, if necessary, process the formula with a succession of fundamental operations. Examples are given at Cox 1891 or at Dunlop 1913.[15] The rearranged fraction in Figure 6 on the right is processed at Dunlop with 8 steps.

$$p = \frac{Wv^2}{2g \times 2240 \times \frac{\pi d^2}{4} \times l}, \qquad p = \frac{Wv^2 \times 1 \times 1 \times 1 \times 1}{16 \times 2240 \times \pi d^2 \times l}.$$

Figure 6: Changing a Formula at Dunlop, page 27

At the same time, the interoperation of scales and cursor now changes the course of a calculation. The cursor has been invented in the middle of the 19th century. The cursor allows new arrangements, new divisions of the scales, as well as the transition between scales, and thus influences the handling of the slide rule. As an example of this, the old scale arrangement (from top to bottom) A=B=C=1-10-100, D=1-10 without cursor changed to the new arrangement A=B=1-10-100, C=D=1-10 with cursor.

Specific collections of calculating examples are still preserved to a limited extent or are transformed into collections of selected exercise examples.

In contrast to completely finished solutions, previous knowledge is now required, both in the understanding of the descriptive formula, as well as in the formula's conversion and processing. One must assume that the improved and deepened training in arithmetic and algebra at schools and technical colleges in the 19th century allowed this change in methodology.

The Definition of Logarithm

Not before the middle of the 19th century did the definition of logarithm serve as an aid for the explanation of the operation of a slide rule. Two explanatory models are applied to make understandable that multiplications or divisions are executed as additions or subtractions of lengths: either the historical conception of a geometric and a corresponding arithmetic progression, or the functional description log (a * b) = log (a) + log (b) and log (a / b) = log (a) - log (b), respectively.

The comparison of the two sequences is the older definition of the logarithm. The definition has been used by John Napier in 1614 and Jost Bürgi in 1620. Cox writes about it:

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2d. Logarithms are a series of
numbers in Arithmetical Progression
(as 0,1,2,3,4, etc.), corresponding
to another series of numbers in
Geometrical Progression (as
1,2,4,8,16, etc.)
We will take two such series and
place them together, thus:-
0  1  2  3  4  5 [...] 10
1  2  4  8  16 32 [...] 1024
Here the first line is a series of
numbers in A.P. and they are the
logarithms of the corresponding
numbers in the second line, which is
a series of numbers in G.P.
[...]
1st. If we add together any two
numbers of the first line as 3 and 5,
their sum 8 corresponds with 256 of
the second line. Now 256 will be
found to be the product of the two
factors 8 and 32, which on the
second line correspond with 3 and 5
on the first line [...] (Cox 1891, S.
1).
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An almost identical explanation is given by Burg 1830. Both focus solely addition or subtraction of logarithms. The question of the existence of a base number is not asked. This is remarkable because the definition of the logarithm $z=b^{\log(z)}$ with the base number b became customary as early as the middle of the 18th century.

Instruction for use within 20th century continue the 19th century method. First they teach basic calculations on the slide rule. In application examples they name the underlying formula, and deduce from it the sequence of the operations on the slide rule. The calculation with logarithms and its implementation on the slide rule is not described in all instructions.

Notes

1. Partridge, Seth: *The Description and Use of an Instrument, called the Double Scale of Proportion,* London, 1661, 1671, 1685, 1692.
2. Reconstruction in van Poelje, Otto E.: *The Sliding Gunter – Reconstruction of the Partridge Scales,* Journal of the Oughtred Society, 16:1, Spring 2007.
3. Other approximations are, more precise, 355/113 at Lalanne, Leon: *Instruction sur les règles à calcul,* 2. Ausg., Paris, 1854 and at other authors. At Cox, William: *The Mannheim and the Duplex Slide Rules,* Keuffel & Esser, New York, 1891 the same value is given with 710/226. Rarely found is 333/106 (*Der logarithmische Rechenschieber und sein Gebrauch,* Albert Nestler, Lahr, 3. Aufl., 1942).
   With the point pi on the scale the approximations with fractions came to an end.
4. Scheffelt, Michael: *Pes Mechanicus Artificialis, Oder Neu-erfundener Maß-Stab,* Ulm, 1718.
5. Scheffelt, 1718, pages 85 and 86. Here he calculates circumference = diameter * 22/7; area = circumference * diameter/4. The numbers are given in the base 12 or duodecimal system, because the length unit is divided into 12 parts: 3 ° 4'7 " ≡ 3 + 4/12 + 7/144
6. Howe, Joseph: *Instructions and a Practical Treatise on the Improved Slide Rule,* London, 1845.
7. The constant 0.433 equals tan(60°)/4 or (sqrt(3/4))/2 and is used in calculating the area of an equilateral triangle.
8. Post-test calculations result in the following relationship: Howe assumes a steam engine with a horizontal beam. On one side of the beam works the force of the piston with diameter D inches applied with a vapor pressure p of 10 or 7 lbs. / square inches. With the same force on the other end of the beam, a piston has to lift a water column with the height of h yards and the diameter d inches.
   The gauge point gp summarizes three variables: the pressure p, the diameter d depending on predefined values from the table plus the specific weight of water relative to the volume in units of inches * inches * yards. This way the problem is reduced to the proportion $D^2 = h * gp$. With the scale arrangement A=B=C=1-10-100 and D=1-10, the scale D directly shows the necessary piston diameter D as a function of h.
   The gauge point is not given with a real quantity, but as a point on the scale with its numerical sequence without a decimal separator.
9. Sedlaczek, Ernest: *Anleitung zum Gebrauche einiger logarithmisch geteilter Rechenschieber,* 2. Aufl., Wien, 1856.
10. Everard, Thomas: *Stereometry made easie, or, The description and use of a new auging-rod or sliding-rule,* London, 1684.
11. Venetsianos, Panagiotis: P*ocketbook of the Gauge Marks,* 2nd edition, The Oughtred Society, 2012.
12. Farey, John: *A Treatise on the Steam Engine,* London, 1827.
   Hoare, Charles: *The Slide Rule and How to Use It,* London, 1868.
   Burg, Adam: *Über die Einrichtung und Anwendung des bei den englischen Mechanikern und Maschinenarbeitern gebräuchlichenSchieberlineals (Sliding Rule*), Jahrbücher des k. k. polytechnischen Institutes in Wien, 16. Bd., Wien, 1830, S. 101-170.
   Schulz v. Straßnicki, Leopold C.: *Anweisung zum Gebrauche des englischen Rechenschiebers,* Wien, 1843.
13. Figure 3 rule 346 (last row) reads C: diameter of sphere – volume of sphere; D: 1.38 – diameter.
   Here the scales C: 1-10-100 and D: 1-10 are used. In addition $1/1.38^2 \approx$ pi/6.
14. The setting in Figure 5 on top should be read as X Inches : Y Centimeter = 26 : 66.
15. Dunlop, H. C. and Jackson, C. S.: *Slide-Rule Notes,* New York, 1913.